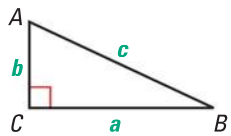
7 CHAPTER SUMMARY

BIG IDEAS — For Your Notebook

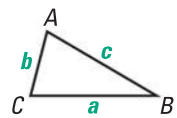
Big Idea 1 — Using the Pythagorean Theorem and Its Converse

The Pythagorean Theorem states that in a right triangle the square of the length of the hypotenuse c is equal to the sum of the squares of the lengths of the legs a and b, so that $c^2 = a^2 + b^2$.

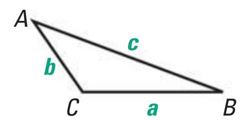
The Converse of the Pythagorean Theorem can be used to determine if a triangle is a right triangle.



If $c^2 = a^2 + b^2$, then $m\angle C = 90°$ and $\triangle ABC$ is a right triangle.



If $c^2 < a^2 + b^2$, then $m\angle C < 90°$ and $\triangle ABC$ is an acute triangle.

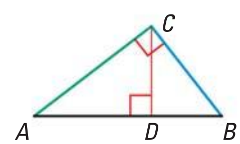


If $c^2 > a^2 + b^2$, then $m\angle C > 90°$ and $\triangle ABC$ is an obtuse triangle.

Big Idea 2 — Using Special Relationships in Right Triangles

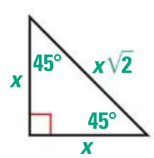
GEOMETRIC MEAN In right $\triangle ABC$, altitude $\overline{CD}$ forms two smaller triangles so that $\triangle CBD \sim \triangle ACD \sim \triangle ABC$.

Also, $\frac{BD}{CD} = \frac{CD}{AD}$, $\frac{AB}{CB} = \frac{CB}{DB}$, and $\frac{AB}{AC} = \frac{AC}{AD}$.



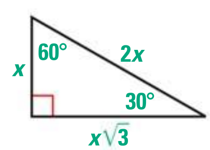
SPECIAL RIGHT TRIANGLES

45°-45°-90° Triangle



hypotenuse = leg • $\sqrt{2}$

30°-60°-90° Triangle



hypotenuse = 2 • shorter leg

longer leg = shorter leg • $\sqrt{3}$

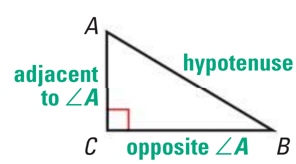
Big Idea 3 — Using Trigonometric Ratios to Solve Right Triangles

The tangent, sine, and cosine ratios can be used to find unknown side lengths and angle measures of right triangles. The values of tan $x°$, sin $x°$, and cos $x°$ depend only on the angle measure and not on the side length.

$\tan A = \frac{\text{opp.}}{\text{adj.}} = \frac{BC}{AC}$ $\qquad \tan^{-1}\frac{BC}{AC} = m\angle A$

$\sin A = \frac{\text{opp.}}{\text{hyp.}} = \frac{BC}{AB}$ $\qquad \sin^{-1}\frac{BC}{AB} = m\angle A$

$\cos A = \frac{\text{adj.}}{\text{hyp.}} = \frac{AC}{AB}$ $\qquad \cos^{-1}\frac{AC}{AB} = m\angle A$



7 CHAPTER REVIEW

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• Multi-Language Glossary
• Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- Pythagorean triple, *p. 435*
- trigonometric ratio, *p. 466*
- tangent, *p. 466*
- sine, *p. 473*
- cosine, *p. 473*
- angle of elevation, *p. 475*
- angle of depression, *p. 475*
- solve a right triangle, *p. 483*
- inverse tangent, *p. 483*
- inverse sine, *p. 483*
- inverse cosine, *p. 483*

VOCABULARY EXERCISES

1. Copy and complete: A Pythagorean triple is a set of three positive integers a, b, and c that satisfy the equation __?__.
2. **WRITING** What does it mean to solve a right triangle? What do you need to know to solve a right triangle?
3. **WRITING** *Describe* the difference between an angle of depression and an angle of elevation.

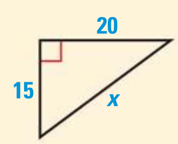
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 7.

7.1 Apply the Pythagorean Theorem *pp. 433–439*

EXAMPLE

Find the value of x.



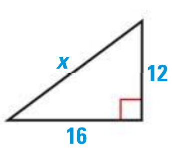
Because x is the length of the hypotenuse of a right triangle, you can use the Pythagorean Theorem to find its value.

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$	**Pythagorean Theorem**
$x^2 = 15^2 + 20^2$	**Substitute.**
$x^2 = 625$	**Simplify.**
$x = 25$	**Find the positive square root.**

EXERCISES

EXAMPLES 1 and 2 on pp. 433–434 for Exs. 4–6

Find the unknown side length x.

4.


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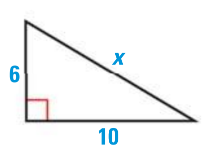
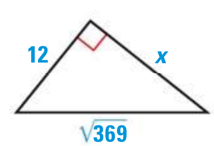
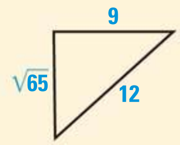


6.


7.2 Use the Converse of the Pythagorean Theorem *pp. 441–447*

EXAMPLE

Tell whether the given triangle is a right triangle.



Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

$$12^2 \stackrel{?}{=} (\sqrt{65})^2 + 9^2$$

$$144 \stackrel{?}{=} 65 + 81$$

$$144 < 146$$

The triangle is not a right triangle. It is an acute triangle.

EXERCISES

EXAMPLE 2 on p. 442 for Exs. 7–12

Classify the triangle formed by the side lengths as *acute*, *right*, or *obtuse*.

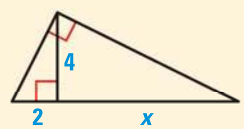
7. 6, 8, 9 **8.** 4, 2, 5 **9.** 10, $2\sqrt{2}$, $6\sqrt{3}$

10. 15, 20, 15 **11.** 3, 3, $3\sqrt{2}$ **12.** 13, 18, $3\sqrt{55}$

7.3 Use Similar Right Triangles *pp. 449–456*

EXAMPLE

Find the value of *x*.



By Theorem 7.6, you know that 4 is the geometric mean of x and 2.

$\frac{x}{4} = \frac{4}{2}$ **Write a proportion.**

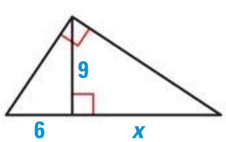
$2x = 16$ **Cross Products Property**

$x = 8$ **Divide.**

EXERCISES

EXAMPLES 2 and 3 on pp. 450–451 for Exs. 13–18

Find the value of *x*.

13.


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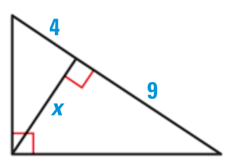

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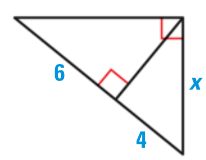

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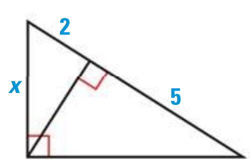

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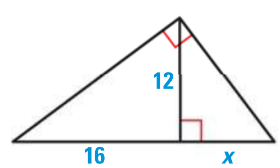
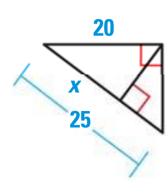


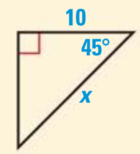
18.


7 CHAPTER REVIEW

7.4 Special Right Triangles *pp. 457–464*

EXAMPLE

Find the length of the hypotenuse.



By the Triangle Sum Theorem, the measure of the third angle must be 45°. Then the triangle is a 45°-45°-90° triangle.

$\text{hypotenuse} = \text{leg} \cdot \sqrt{2}$ **45°-45°-90° Triangle Theorem**

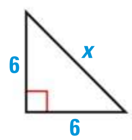
$x = 10\sqrt{2}$ **Substitute.**

EXERCISES

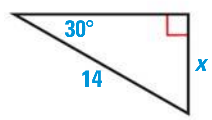
EXAMPLES 1, 2, and 5 on pp. 457–459 for Exs. 19–21

Find the value of *x*. Write your answer in simplest radical form.

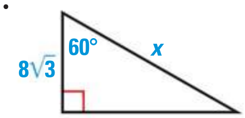
19.



20.



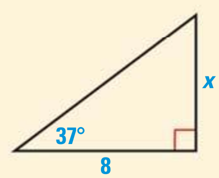
21.



7.5 Apply the Tangent Ratio *pp. 466–472*

EXAMPLE

Find the value of *x*.



$\tan 37° = \frac{\text{opp.}}{\text{adj.}}$ **Write ratio for tangent of 37°.**

$\tan 37° = \frac{x}{8}$ **Substitute.**

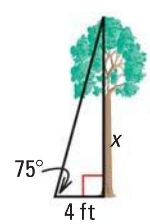
$8 \cdot \tan 37° = x$ **Multiply each side by 8.**

$6 \approx x$ **Use a calculator to simplify.**

EXERCISES

EXAMPLE 2 on p. 467 for Exs. 22–26

In Exercises 22 and 23, use the diagram.

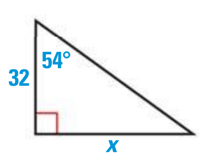


22. The angle between the bottom of a fence and the top of a tree is 75°. The tree is 4 feet from the fence. How tall is the tree? Round your answer to the nearest foot.

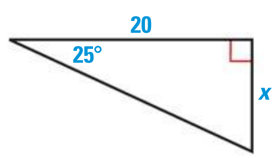
23. In Exercise 22, how tall is the tree if the angle is 55°?

Find the value of *x* to the nearest tenth.

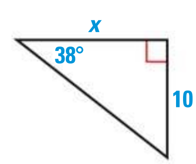
24.



25.



26.

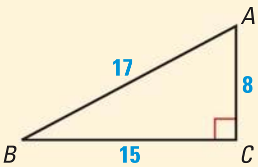


7.6 Apply the Sine and Cosine Ratios

pp. 473–480

EXAMPLE

Find sin A and sin B.



$$\sin A = \frac{\text{opp.}}{\text{hyp.}} = \frac{BC}{BA} = \frac{15}{17} \approx 0.8824$$

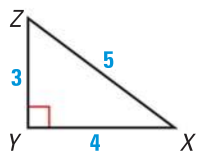
$$\sin B = \frac{\text{opp.}}{\text{hyp.}} = \frac{AC}{AB} = \frac{8}{17} \approx 0.4706$$

EXERCISES

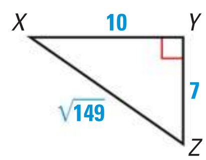
EXAMPLES 1 and 2
on pp. 473–474
for Exs. 27–29

Find sin X and cos X. Write each answer as a fraction, and as a decimal. Round to four decimals places, if necessary.

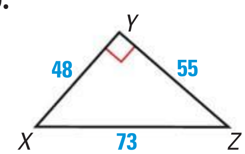
27.



28.



29.

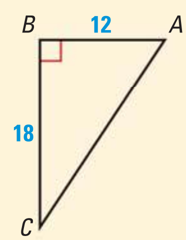


7.7 Solve Right Triangles

pp. 483–489

EXAMPLE

Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.



Because $\tan A = \frac{18}{12} = \frac{3}{2} = 1.5$, $\tan^{-1} 1.5 = m\angle A$.

Use a calculator to evaluate this expression.

$\tan^{-1} 1.5 \approx 56.3099324\ldots$

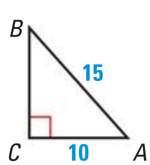
So, the measure of $\angle A$ is approximately 56.3°.

EXERCISES

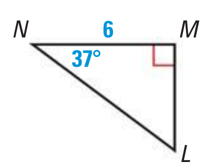
EXAMPLE 3
on p. 484
for Exs. 30–33

Solve the right triangle. Round decimal answers to the nearest tenth.

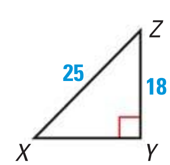
30.



31.



32.



33. Find the measures of $\angle GED$, $\angle GEF$, and $\angle EFG$. Find the lengths of $\overline{EG}$, $\overline{DF}$, $\overline{EF}$.